Equations and Conversion Factors for Chapters 16, 17, & 18

Note: the curvy volume symbol in the book is not in a standard font; below, V = velocity and V = volume.

Steady Heat Conduction, Convection, & Radiation in Walls, Cylinders, & Spheres

$$\alpha = \frac{k}{\rho c_p}$$

$$\dot{Q}_{cond} = k A \frac{\Delta T}{L} = \frac{T_1 - T_2}{R_{wall}}$$

$$\dot{Q}_{conv} = h A (T_s - T_\infty) = \frac{(T_s - T_\infty)}{R_{conv}}$$

$$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4$$

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

$$\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty)$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$\dot{Q}_{cond,sphere} = \frac{T_1 - T_2}{R_{sphere}}$$

$$\Delta T = \dot{Q} R$$

$$R_{wall} = \frac{L}{k A}$$

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2 \pi L k}$$

$$R_{sphere} = \frac{r_2 - r_1}{4 \pi r_1 r_2 k}$$

$$R_{conv} = \frac{1}{h A_s}$$

$$R_{interface} = \frac{1}{h_c A} = \frac{R_c}{A}$$

$$R_{rad} = \frac{1}{h_{rad} A_s}$$

Series: $R_{total} = R_1 + R_2 + R_3 + \ldots$

Parallel: $\dfrac{1}{R_{total}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots$

$$h_{combined} = h_{conv} + h_{rad}$$

$$W = m g$$

$$A_{circle} = \frac{\pi}{4} d^2$$

Circumference of a circle $= \pi d$

$$A_{sphere} = \pi d^2$$

$$V_{sphere} = \frac{\pi d^3}{6}$$

Fins

Very long fin:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty)$$

$$m = \sqrt{\frac{h p}{k A_c}}$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh m L}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty) \tanh m L$$

$$L_c = L + \frac{A_c}{p}$$

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max.}}$$

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no fin}}$$

Lumped System Analysis

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{h A_s}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$\text{Bi} = \frac{h L_c}{k} < 0.1$$

$$L_c = \frac{V}{A_s}$$

$$\dot{Q} = h A_s [T(t) - T_i]$$

$$Q = m c_p [T(t) - T_i]$$

$$Q_{max} = m c_p [T_\infty - T_i]$$

Transient Heat Conduction in Walls, Cylinders, Spheres, & Semi-infinite Solids

$\tau = \dfrac{\alpha t}{L^2}$ or $\dfrac{\alpha t}{r_o^2}$	$\theta_{wall} = \dfrac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\dfrac{\lambda_1 x}{L}\right),\ \tau > 0.2$
	$\theta_{cyl} = \dfrac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0\left(\lambda_1 r/r_o\right),\ \tau > 0.2$
	$\theta_{sphere} = \dfrac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \dfrac{\sin\left(\lambda_1 r/r_o\right)}{\left(\lambda_1 r/r_o\right)},\ \tau > 0.2$

Constant surface temperature $T_s = $ constant	$\dfrac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)$ and $\dot{q}_s(t) = \dfrac{k(T_s - T_i)}{\pi \alpha t}$
Constant heat flux $\dot{q}_s = $ constant	$T(x,t) - T_i = \dfrac{\dot{q}_s}{k}\left[\sqrt{\dfrac{4\alpha t}{\pi}}\exp\left(\dfrac{-x^2}{4\alpha t}\right) \text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)\right]$
Convection on the surface $\dot{q}_s(t) = h[T_\infty - T(0,t)]$	$\dfrac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\dfrac{hx}{k} + \dfrac{h^2 \alpha t}{k^2}\right) \text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}} + \dfrac{h\sqrt{\alpha t}}{k}\right)$
Energy pulse at the surface $e_s = $ constant	$T(x,t) - T_i = \dfrac{e_s}{k\sqrt{\pi t/\alpha}}\exp\left(\dfrac{-x^2}{4\alpha t}\right)$
Semi-infinite solids	$T_s = \dfrac{\sqrt{(k\rho c_p)_A}\,T_{A,i} + \sqrt{(k\rho c_p)_B}\,T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}}$

Metric Prefixes, Constants, and Unit Conversions

n	=	nano-	=	10^{-9}	$\rho_{water} = 1000\ \dfrac{\text{kg}}{\text{m}^3}$		Pa	=	$\dfrac{\text{N}}{\text{m}^2}$	
μ	=	micro-	=	10^{-6}						
m	=	milli-	=	10^{-3}	$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$		N	=	$\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}$	
c	=	centi-	=	10^{-2}						
k	=	kilo-	=	10^{3}	$P_0 = 1\,\text{atm} = 101\,\text{kPa}$		W	=	$\dfrac{\text{N}\cdot\text{m}}{\text{s}}$ = $\dfrac{\text{J}}{\text{s}}$	
M	=	mega-	=	10^{6}						
G	=	giga-	=	10^{9}	$\sigma = 5.6704 \times 10^{-8}\ \text{W}/\text{m}^2\text{K}^4$		J	=	N·m = W·s	
T	=	tera-	=	10^{12}						
					$°\text{C} + 273 = \text{K}$					

Types of Problems

- Steady-state heat transfer through a plane wall (many variations)

- Steady-state heat transfer through the wall of a cylinder or sphere

- Fins

- Transient heat conduction: lumped system analysis, Heisler charts, semi-infinite solids, multidimensional systems

- General knowledge: types of heat transfer, terminology (k, α, h, etc.)